HEAT EXCHANGER PERFORMANCE RANKING

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by

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ABSTRACT

Traditionally data for plate-finned surfaces have been presented in terms of heat transfer coefficients and friction factors referred to the exposed area, as a function of Reynolds number based on the minimum free flow area. This method of data presentation does not permit comparison of surfaces in any simple manner.

Soland[1] proposed a method of surface comparison where heat transfer coefficient and friction factor is referred to the base area and Reynolds number is based on open flow area, as though the fins were not present.

Soland's method is applied to practical heat exchanger design problems and the usefullness of his method is evaluated. Numerous surfaces not examined by Soland are evaluated. Based on the four comparison criteria considered, these newly evaluated surfaces are compared with Soland's results.

Appendix I provides a method for sizing Cross Flow Plate-Finned Heat Exchangers.

Thesis Supervisor: Professor Warren M. Rohsenow
Title: Professor of Mechanical Engineering

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NOMENCLATURE

Symbol	Definition	Units
A_b	heat transfer area of base surface ignoring any enhancement; equals length times heated perimeter	ft^2
A_c	minimum free flow area	ft^2
A_f	frontal area of heat exchanger core	ft^2
A_{fin}	fin or extended area	ft^2
A_F	flow area ignoring any enhancing surfaces	ft^2
A_T	total heat transfer area	ft^2
b	plate spacing	ft
c_p	specific heat	$BTU/lbm\text{-}^oF$
C	flow stream capacity rate ($w\ c_p$)	$BTU/hr\text{-}^oF$
D_n	nominal diameter; defined by (1b)	ft
f	friction factor based on total area (A_T); defined by (4a)	——
f_n	friction factor based on base area (A_b); defined by (4b)	——
g_o	32.174 $lbm\text{-}ft/lbf\text{-}sec^2$	——
G_c	mass flux based on minimum free flow area; defined by (2a)	$lbm/hr\text{-}ft^2$
G_n	mass flux based on free flow area (A_F); defined by (2b)	$lbm/hr\text{-}ft^2$
h	heat transfer coefficient based on total area (A_T); defined by (5a)	$BTU/hr\text{-}ft^2\text{-}^oF$

h_n	heat transfer coefficient based on base area (A_b); defined by (5b)	BTU/hr-ft^2-^{o}F
j	Colburn j-Factor based on total area (A_T); defined by (7a)	- -
j_n	Colburn j-Factor based on base area (A_b); defined by (7b)	- -
j_s	Colburn j-Factor for smooth surface; defined by (27)	- -
k	thermal conductivity	BTU/hr-ft-^{o}F
ℓ	fin length from root to center (=b/2)	ft
L	heat exchanger length	ft
m	component of fin efficiency (η_f) ; defined by (10)	- -
NTU	number of transfer units; defined by (20)	- -
p	pressure	lbf/ft^2
P	pumping power	hp
q	heat transfer rate	BTU/hr
q/A	heat flux	BTU/hr-ft^2
r_h	hydraulic radius, defined by (1a)	ft
T	temperature	^{o}F
U	overall heat transfer coefficient	BTU/hr-ft^2-^{o}F
V	heat exchanger volume on one side	ft^3
w	mass flow rate	lbm/hr.
X,Y,Z	principal dimensions of heat exchanger	ft

DIMENSIONLESS GROUPS

Nu	Nusselt number; defined by (6a)	- -
Nu_n	Nusselt number; defined by (6b)	- -
Pr	Prandtl number	- -
Re	Reynolds number based on minimum free flow area (A_c); defined by (3a)	- -
Re_n	Reynolds number based on free flow area (A_F); defined by (3b)	- -

SUBSCRIPTS

a	case a parameter (Shape, V = const.)	- -
b	case b parameter (P, V = const.)	- -
c	case c parameter (NTU, P = const.)	- -
d	case d parameter (NTU, V = const.)	- -
e	enhanced surface	- -
m	heat exchanger metal	- -
s	smooth surface	- -

MISCELLANEOUS

α	ratio of total heat transfer area of one side of the exchanger to <u>total</u> exchanger volume	ft^{-1}
β	ratio of total heat transfer area (A_T) to volume (V)	ft^{-1}
ΔP_F	core friction pressure drop	lbf/ft^2
η_f	fin efficiency; defined by (9)	- -
η_o	total surface temperature effectiveness; defined by (8)	- -

μ	viscosity	lbm/hr-ft
ρ	density	lbm/ft^3
ε	heat exchanger effectiveness	- -
δ	fin thickness	ft
ψ	pin diameter	ft
σ	ratio of free flow area to frontal area (A_c/A_f)	- -

LIST OF FIGURES

I. INTRODUCTION

A. Purpose

The designer of a heat exchanger has to select the heat exchanger surfaces for his design. In order to enable the designer to select the optimum surfaces, a logical, accurate, and easily used technique should be employed to make meaningful comparison among candidate surfaces. Soland[1] proposed a method of comparison that appears to permit such comparisons among surfaces, and he performed this comparitive analysis on the surfaces found in Kays and London[2] to determine the "best" surface, for the cases considered.

The purpose of this paper is to: (1) identify the need for a method of comparison among various candidate heat transfer surfaces; (2) apply Soland's method to heat exchanger design problems and evaluate his method's effectiveness; and (3) to compare data from more recent surfaces with that of Kays and London[2].

B. Background

Heat exchangers are critical elements in energy extraction and recovery systems. Applications include gas turbine plants, aircraft cooling, electronics cooling, marine propulsion plant condensers and automobile cooling systems, to name but a few. These applications involve gas-to-gas, liquid-to-liquid, or gas-liquid service.

Prior to 1945, the only generally available data on heat transfer and flow friction characteristics of heat transfer surfaces was for simpler geometries[2]. With the development of more complex aircraft and the increased complexity (and heat generation) of electronics equipments, the need for lighter weight and smaller size heat exchangers was indicated. Gas turbine plant heat exchanger design provided the incentive to investigate the construction and testing of surfaces for compact heat exchanger design.

Many of the surfaces considered were of the "plate-fin" variety with one of the following surface geometries: plain fins, louvered fins, strip fins, wavy fins, and pin fins. Some other type surfaces were also investigated, such as screen matrices, sphere matrix, finned tubes, and later perforated enhanced surfaces.

Optimum heat exchanger design often means transferring a given amount of heat at the lowest "cost". "Cost" may mean capital costs to fabricate the heat exchanger system plus operating costs to pump the heat transfer fluid, or "cost" may mean heat exchanger weight and/or volume, as in aircraft and other mobile applications. It has been shown that tubular heat exchangers have surface-to-volume ratios (β) ranging from 20-100 ft^2/ft^3, while plate-fin heat exchangers have β's of 200-1800 ft^2/ft^3 [3]. While finned surfaces usually have lower heat transfer coefficients, they are compensated by larger surface areas with a net improvement in heat transfer.

Design and testing of various enhanced surfaces has continued since 1945 until the present; indeed, reference [4] was completed in 1971 after 24 years of work at Stanford University! The results of some of this testing will be discussed later.

C. Surface Testing and Data Presentation

In order to provide heat transfer and flow friction data on a given plate-fin heat transfer surface, experimental testing of the surface is required because, except for the very simplest of surfaces, theoretical predictions of performance for proposed new surface designs has not been at all accurate due to the complex interactions involved.

The general method of testing plate-fin surfaces has varied little over the past thirty years. An experimental facility is constructed to allow insertion of the heat exchanger core sample to be tested. The heat source side is usually either hot water[5] or condensing steam[2], the secondary side of the experimental facility is basically an air wind tunnel with heaters or another heat exchanger to allow proper temperature controls of the air into the test core. A system of thermocouples, pressure detectors, and flow measuring devices is included and testing is conducted at various flow rates and heat transfer rates. Another method of testing is called the "Single-Blow" method, and is described in reference [6].

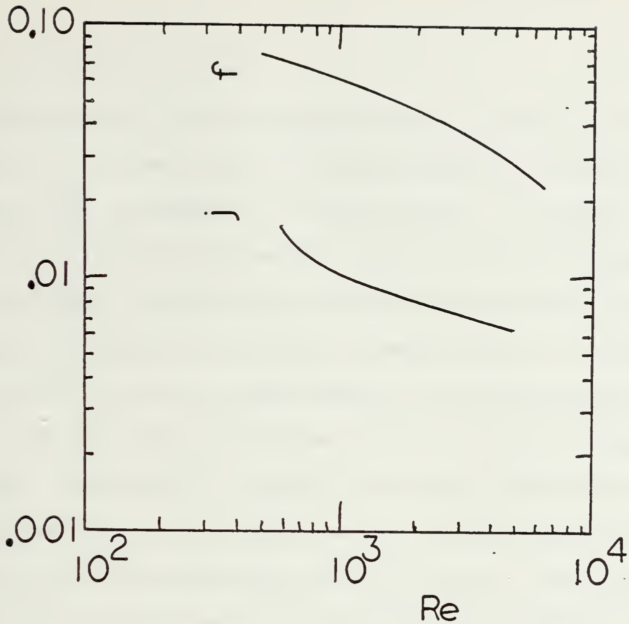
Whatever the technique, the results include, Fanning friction factor f, Prandtl number, and Reynolds number and Colburn j-factor

($j = \frac{h}{G\,c_p}\,Pr^{2/3}$). These nondimensional numbers contain the heat transfer and flow friction characteristics of the heat transfer core surface under investigation. Figure 1 is an example of the manner in which this information is presented in the literature.

D. The Problem Facing the Designer

When the designer of a heat exchanger has determined what his constraints are in the area of: allowed pressure loss; temperature change; amount of heat to be transferred; heat exchanger weight, volume, and fouling and corrosion considerations; configuration; materials and fabrication capabilities; etc., he is ready to start his selection of heat transfer surfaces to be employed in his final exchanger design. The designer has two possible paths to follow: he can choose to design a surface himself or he can choose to examine previously designed surfaces for which test results are available in the literature.

If the heat exchanger designer decides to design his own surface, he will require that the surface be tested to determine its heat transfer and flow friction characteristics as described in section I.C. This testing will involve added expense and implies going to the literature to examine results on existing surfaces to aid in the design of his surface. Existing data may help him decide what general type surface he will utilize, should the fins be thicker or thinner than existing surface, more or less fins per inch, greater



Fin Pitch - 16.3 per inch

Plate Spacing - b = .253 in.

Flow passage hydraulic diameter - 4 r_h = 0.00657 ft.

Fin metal thickness - .006 in.

Total heat transfer area/volume between plates - β = 475 ft^2/ft^3

Fin area/total area - .890

Figure 1. An Example of the Traditional Method of Presenting Required Data and Geometrical Properties.

or lesser plate spacing, etc.

Let us assume that the designer decides to utilize an existing surface for which data exist, or having designed and tested his own surface wishes to compare it to other surfaces. The designer searches the literature and selects a great number of candidates for his design. How does he now compare these surfaces to determine which will be selected for his heat exchanger design? The designer will be able to rule out a great number of the candidate surfaces due to incomplete data presentation. In 1964, Battelle Memorial Institute, reference [7], conducted a literature search and reported, "It has been noted that, of more than 200 references examined, adequate data are presented from only 25 of them... The primary reasons that more data were not found useful were either that the thermodynamic performance data or descriptions of the heat-exchanger surface geometry were incomplete." This situation continues to exist, be it because of a manufacturer's reluctance to share his "secrets" or the scientist's wish to make an academic point and neglects to include data in his report which would allow consideration of the tested surfaces for a practical application.

Of the surfaces for which complete information and data are available, the designer still has to decide which is the optimum surface for his heat-exchanger design. Figure 2 is an example of data for but two surfaces. Surface A has the better heat transfer

characteristic at a given Reynolds number but it also has a higher friction factor at that same Reynolds number. Which surface is better suited to the heat-exchanger design when all of the design constraints are considered?

The fact that there is not an obvious method to compare surfaces A and B has prompted methods of comparisons to be developed [5], [8], [9] and it shall be the purpose of this paper to examine the previously mentioned Soland's method of comparison.

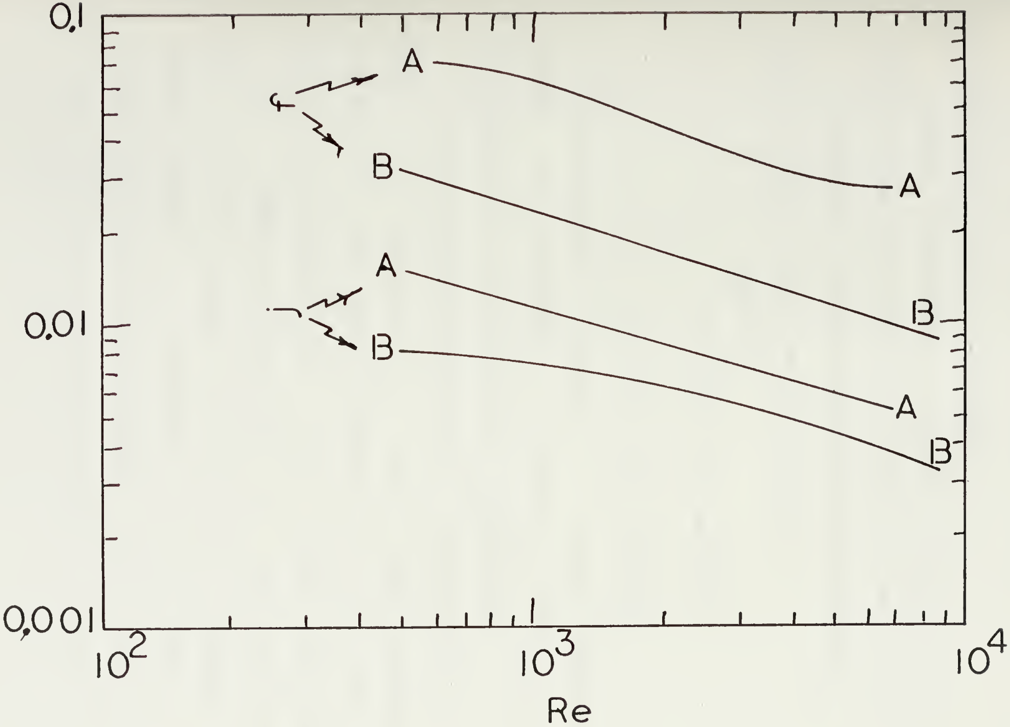


Figure 2. Traditional Method of Presenting f and j vs. Re , Showing Difficulty in Comparing Surfaces A and B.

II. PROPOSED COMPARISON TECHNIQUE

A. Derivation of Basis of Comparison

Soland, in reference [1], provides a detailed derivation of his proposed comparison technique, which will be summarized here.

Performance of various finned and unfinned surfaces is compared with the following quantities held constant:

1. w , flow rate
2. $T_{h, in}$, hot fluid inlet temperature
3. $T_{c, in}$, cold fluid inlet temperature

The performance of only one-side of the heat exchanger is considered. This is equivalent to considering the controlling heat transfer resistance to be on the side under consideration.

The data found in the literature is presented with h and f based on total exposed area, A_T , and Re based on minimum free flow area, A_c , and a hydraulic diameter of the actual flow passage. Soland's method converts these h and f values to new quantities h_n and f_n based on base plate area, A_b , and a new Reynolds number, Re_n , is calculated based on the flow area ignoring any enhancing surfaces, A_F. The effects of the fins is taken as an increased heat flux and hence increased h on the base plate area. In order to be able to incorporate the effect of the fins into the h_n , the metal conductivity of the fins must be specified. Table I

TABLE I. DEFINITIONS. (Reference 1)

Quantity	Commonly Used		Proposed	
hydraulic diameter or radius	$r_h \equiv \frac{A_c\ L}{A_T}$	(1a)	$D_n \equiv \frac{4\ A_F\ L}{A_b} = \frac{4\ V}{A_b}$	(1b)
mass velocity	$G_c \equiv \frac{w}{A_c}$	(2a)	$G_n \equiv \frac{w}{A_F}$	(2b)
Reynolds number	$Re \equiv \frac{4\ G_c\ r_h}{\mu}$	(3a)	$Re_n \equiv \frac{G_n\ D_n}{\mu}$	(3b)
friction factor	$f \equiv \frac{\Delta p_F}{\frac{L}{r_h} \frac{G_c^2}{2\ \rho\ g_o}}$	(4a)	$f_n \equiv \frac{\Delta p_f}{4 \frac{L}{D_n} \frac{G_n^2}{2\ \rho\ g_o}}$	(4b)
heat transfer coefficient	$h \equiv \frac{q/\eta_o\ A_T}{\Delta T}$ *	(5a)	$h_n \equiv \frac{q/A_b}{\Delta T}$	(5b)
Nusselt number	$Nu \equiv \frac{4\ r_h\ h}{k}$	(6a)	$Nu_n \equiv \frac{h_n\ D_n}{k}$	(6b)

TABLE I. DEFINITIONS. (continued)

Quantity	Commonly used	Proposed
Colburn j	$j \equiv \frac{h}{G_c \; c_p}(Pr)^{2/3}$ (7a)	$j_n \equiv \frac{h_n}{G_n \; c_p}(Pr)^{2/3}$ (7b)

$$* \; \eta_o \equiv 1 \frac{A_{fin}}{A_T}(1 - \eta_F) \quad (8)$$

$$\eta_f \equiv \frac{\tanh m\ell}{m\ell} \quad (9)$$

$$m \equiv \sqrt{\frac{2}{\delta}\;\frac{h}{k_m}} \quad \text{(10a) thin sheet fins}$$

$$m \equiv \sqrt{\frac{4}{\psi}\;\frac{h}{k_m}} \quad \text{(10b) circular pin fins}$$

presents the proposed definitions in the technique and also shows the definitions commonly used in the literature.

To convert data found in the literature to the new basis, equations (1) through (10) are used to establish the following ratios.

$$\frac{A_b}{A_T} = \frac{2\ a\ L}{\beta\ V} = \frac{2}{\beta\ b} \tag{11}$$

where

$\beta = A_T/V$

$$\frac{A_F}{A_c} = \frac{L\ a\ b}{A_T\ r_h} = \frac{1}{\beta\ r_h} \tag{12}$$

$$\frac{G_n}{G_c} = \frac{A_c}{A_F} = \beta\ r_h \tag{13}$$

$$\frac{Re_n}{Re} = \frac{D_n\ G_n}{4\ r_h\ G_c} = \frac{\beta\ b}{2} \tag{14}$$

$$\frac{f_n}{f} = \frac{A_F\ A_T\ G_c^{\ 2}}{A_c\ A_b\ G_n^{\ 2}} = \frac{b}{2\ \beta^2\ r_h^{\ 3}} \tag{15}$$

$$\frac{j_n}{j} = \frac{h_n\ G_c}{h\ G_n} = \frac{\eta_o\ b}{2\ r_h} \tag{16}$$

In order to solve for η_o , the material used to construct the heat exchanger and the gas must be specified. In this paper,

aluminum ($k \approx$ 100 BTU/ft-hr-$^{\circ}$F) and air at 90°F will be assumed unless otherwise specified. Figure 3 shows an example of data presented on both basis. Additional curves of j_n vs Re_n could be drawn for other magnitudes of the thermal conductivity of the fins.

For any heat exchanger, power per unit volume on one side is:

$$\frac{P}{V} = \frac{w\ \Delta P_f}{\rho\ V} = \left(\frac{2\ \mu^3}{g_o\ \rho^2}\right)\left(\frac{f_n\ Re_n^3}{D_n^4}\right) \tag{17}$$

For a given fluid, holding temperature constant:

$$\frac{P}{V} \propto \frac{f_n\ Re_n^3}{D_n^4} \tag{18}$$

For any heat exchanger:

$$q = \varepsilon\ (T_{h,in} - T_{c,in})\ w\ c_p \tag{19}$$

NTU is defined as:

$$NTU \equiv \frac{A\ h_n}{w\ c_p} \tag{20}$$

It is noted that the relationship between ε and NTU is always monotonically increasing and an increase in $A\ h_n$ causes an increase in NTU which means ε and thus q are greater.

From (20) and Table I:

$$\frac{NTU}{V} = \frac{A\ h_n}{V\ w\ c_p} = \frac{4\ \mu}{Pr^{2/3}}\ \frac{j_n\ Re_n}{w\ D_n^2} \tag{21}$$

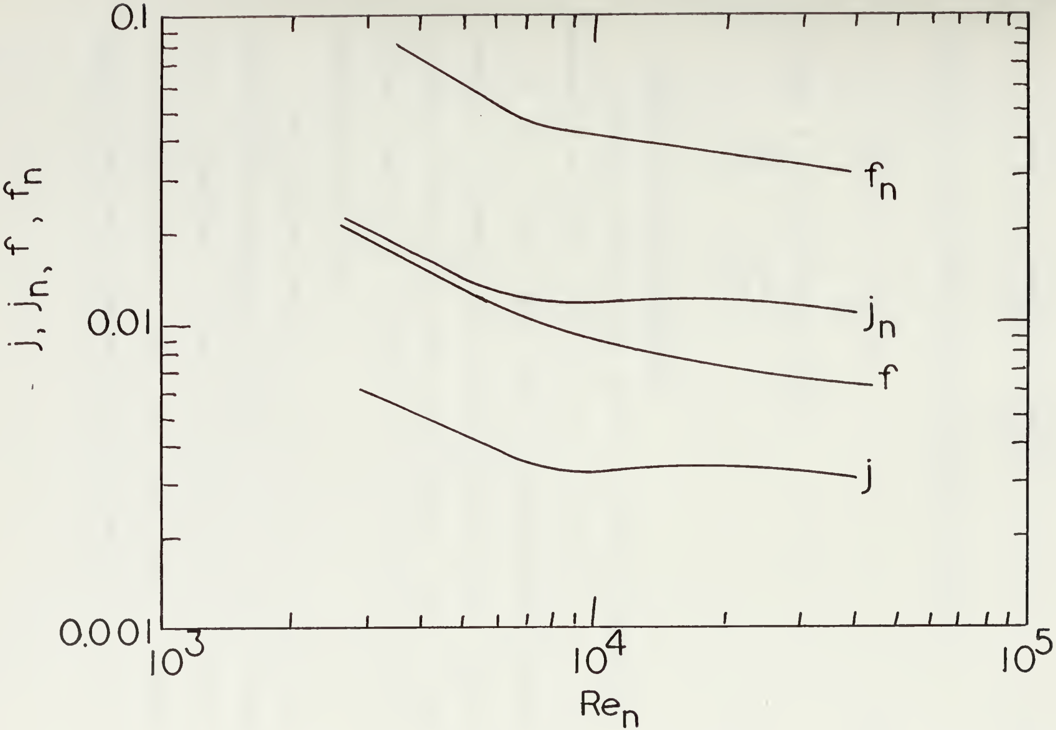


Figure 3. Comparison of Colburn j Factors (j and j_n), and Comparison of Friction Factors (f and f_n).

or,

$$\frac{A\,h_n}{V} = \left(\frac{4\,c_p\,\mu}{Pr^{2/3}}\right)\left(\frac{j_n\,Re_n}{D_n^{\,2}}\right) \tag{22}$$

Again for a given fluid, holding temperature constant:

$$\frac{A\,h_n}{V} \propto \frac{j_n\,Re_n}{D_n^{\,2}} \tag{23}$$

In that w is held constant, also:

$$\frac{NTU}{V} \propto \frac{j_n\,Re_n}{D_n^{\,2}} \tag{24}$$

B. Method of Surface Comparison

Equations (18) and (24) provide us with the performance parameters we desire. With the data in the form f_n vs. Re_n and j_n vs. Re_n it is a simple matter to calculate the performance parameters of equations (18) and (24) and plot them. Figure 4 is an example of such a plot where two surfaces, 1 and 2 , have been plotted to show how a determination of heat-exchanger relative performance may be made.

Four different comparisons are immediately available from Figure 4 and are indicated by points a , b , c , and d on surface 2. Point o on surface 1 represents the reference heat-exchanger design to which each of the four points on surface 2 will be compared.

Point a: Same heat-exchanger shape and volume ($L_a = L_o$, $V_a = V_o$,

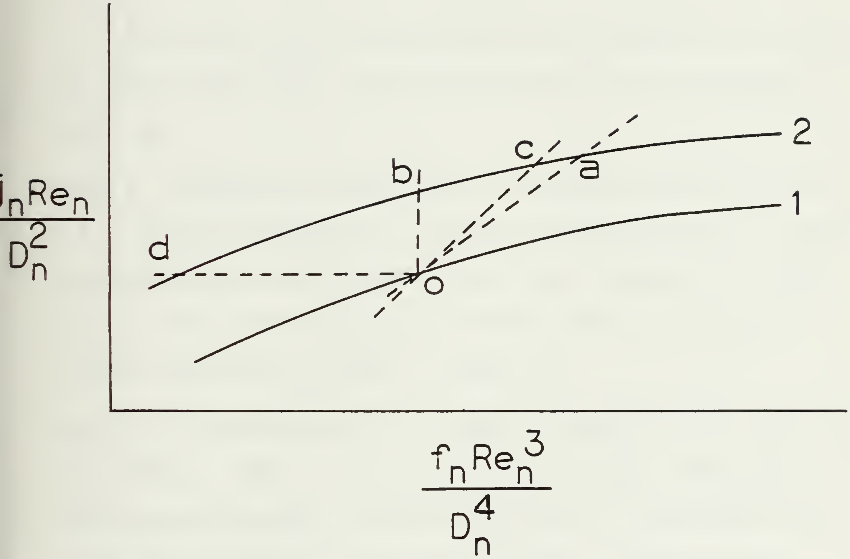


Figure 4. Performance Parameter Curves for Two Surfaces Showing Points Used in Sample Comparisons.

$A_{F_a} = A_{F_o}$). Because w and A_F are fixed:

$$Re_{n_a} = Re_{n_o} \times \frac{D_{n_a}}{D_{n_o}} \qquad (25)$$

The results of this comparison are easily obtained as the ratios of ordinate values $\frac{A_{h_n}}{V}$ and abscissa values $\frac{P}{V}$ and are shown in Figure 5a.

Point b. Same heat-exchanger volume and pumping power ($V_b = V_o$, $P_b = P_o$). Point b is located on a vertical line through point o because pumping power per unit volume is equal in both exchangers. The NTU ratio of the two heat exchangers is obtained simply as the ratio ordinate values, and are shown in Figure 5b.

Point c. Same pumping power and number of transfer units. ($P_c = P_o$, $NTU_c = NTU_o$). Point c is located on a line having a slope equal to unity and through point o because both NTU and P are constant and each axis is inversely proportional to volume. The ratio of the volume required using surface 2 to the volume required using surface 1 is simply the ratio of either ordinates or abscissas at points c and o. Figure 5c shows the result of this comparison .

Case d: Same volume and number of transfer units. ($V_d = V_o$, $NTU_d = NTU_o$). Point d is located on a horizontal line through point o because NTU/V is constant. The ratio of pumping power required by surface 2 and surface 1 is the ratio of abscissas and

and Figure 5d shows the decreased pumping power required by surface 2 as a function of Reynolds number.

Reference 1 provides further details as to how shape will change in the above four comparisons and performs such comparisons on most of the surfaces found in reference (2).

It may be noted that when a plot such as Figure 4 is constructed, the higher the curve lies the better the surface for each of the four cases investicated. The next section of this report shall evaluate this comparison method using a practical example.

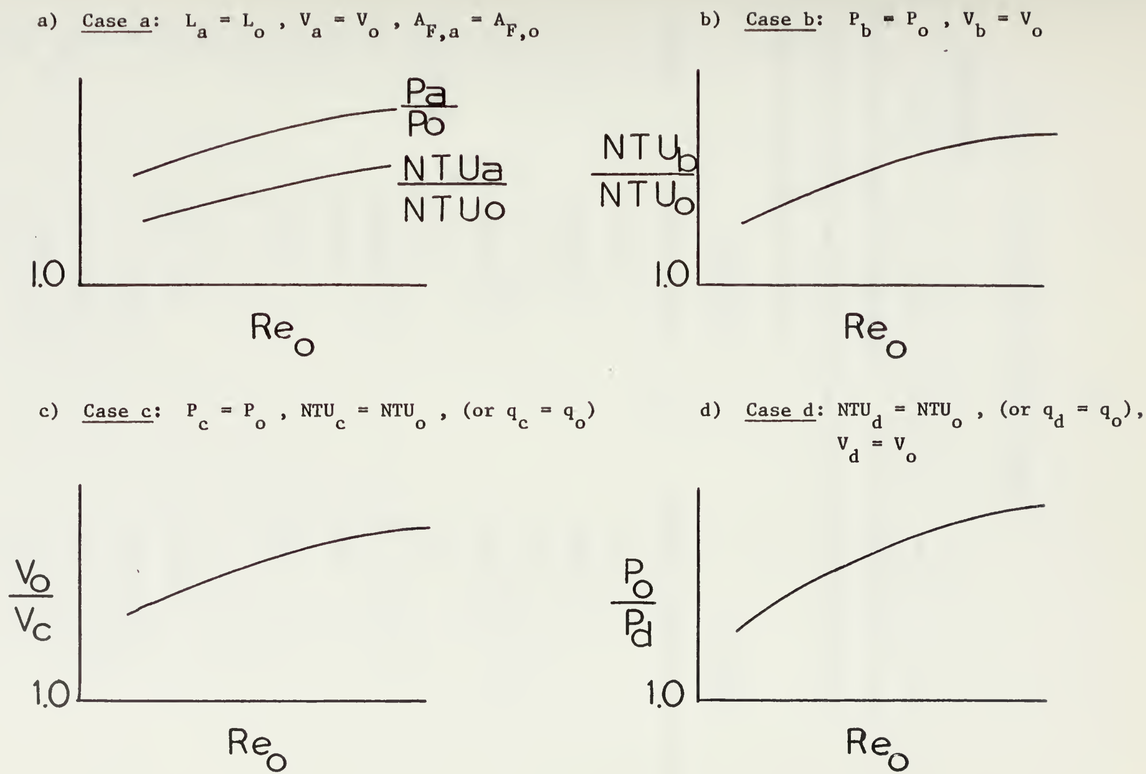


Figure 5. Typical Performance Comparison Results.

III. HEAT EXCHANGER DESIGN PROBLEM

A. Description.

Appendix I is a procedure which can be used for sizing cross-flow plate-finned heat exchangers. The following data, taken from reference [2], is used to determine the required heat-exchanger size for the given conditions.

	Gas Side	Air Side
SURFACE	PLAIN PLATE-FIN 11.1	LOUVERED PLATE-FIN 3/8-6.06
b	.25 in.	.25 in.
r_h	.00253 ft	.00365 ft
δ	.006 in.	.006 in.
β	.367 ft^2/ft^3	256 ft^2/ft^3
A_{fin}/A_T	.756	.640
w	195,895 lb/hr	193,000 lb/hr
T_{in}	805^oF	347^oF
T_{out}	477^oF	691^oF
ΔP	.42 psi	.54 psi
P_{in}	14.9 psi	132 psi
μ	.073 lb/hr-ft	.069 lb/hr-ft
C_{pm}	.259 BTU/lb-^oF	.251 BTU/lb-^oF
ρ_m	.0362 lb/ft^3	.3565 lb/ft^3
Pr	.67	.67
k	- 12 BTU/(hr-ft^2-^oF/ft)	-
a	.012 in.	-

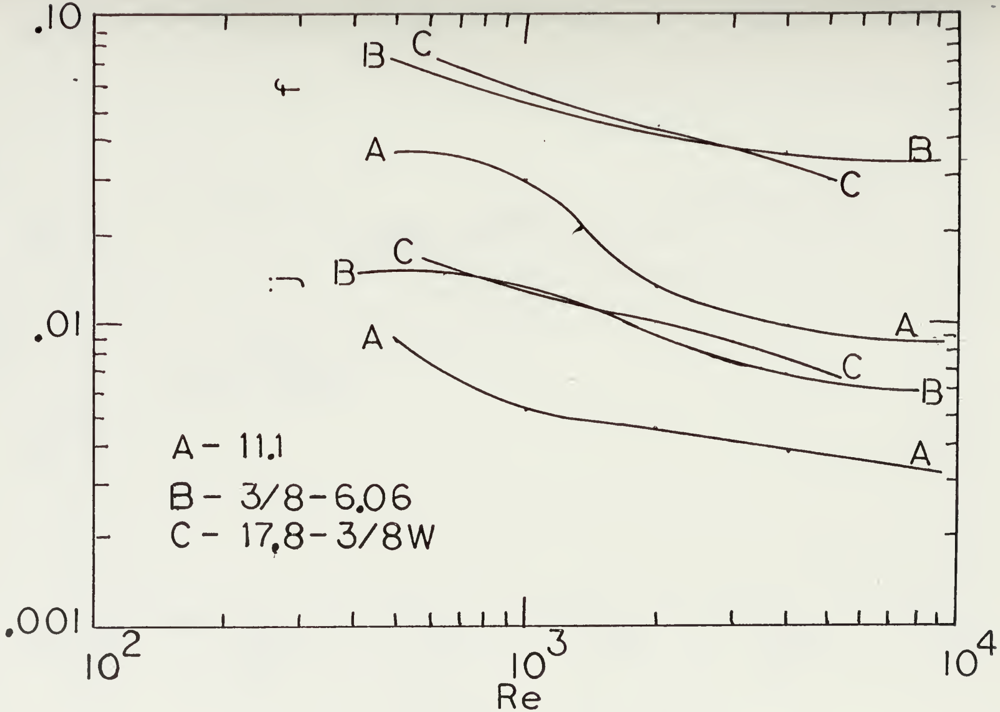


Figure 6. Data for Three Different Surfaces Considered In Example Problem.

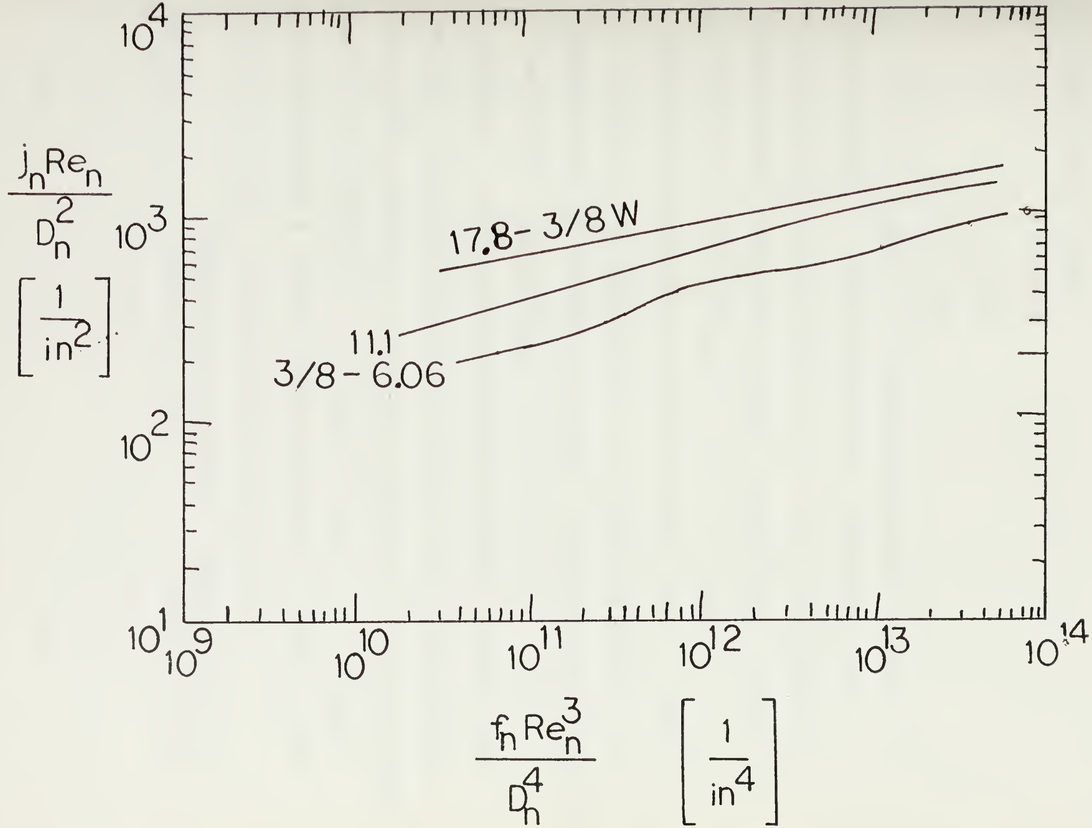


Figure 7. Performance Parameter Plot for Example Problem.

Appendix II shows the calculations involved. Results for the principal dimensions are:

X = 6.0 ft

Y = 3.0 ft

Z = 7.5 ft

This is a possible heat-exchanger design that satisfies the given conditions. If the gas side surface (Plain Plate-Fin 11.1) were replaced with Wavy-Fin Surface 17.8 - 3/8 W of reference [2] would this allow us to build a smaller heat-exchanger? If both the gas side and air side surfaces were replaced with the 17.8 - 3/8 W surface would this permit an even smaller design? The f and j vs. Re data for the three surfaces are presented in Figure 6. The designer is not able to make a meaningful comparison based on inspection of these curves. The problems presented correspond to point c on Figure 4. In that the controlling heat transfer is not on only one side of the heat-exchanger, the predicted volume reduction will not be fully realized but a meaningful comparison of the three surfaces may be made by plotting the performance parameter curves. Figure 7 shows that the 17.8 - 3/8 W surface is superior to either of the other two surfaces. Using the procedure of Appendix I:

Original	Replace Gas Side Surface with 17.8 - 3/8 W	Replace Both Sides with 17.8 - 3/8 W
X = 6.0 ft	X = 3.8 ft	X = 2.86
Y = 3.0 ft	Y = 1.3 ft	Y = 1.02
Z = 7.5 ft	Z = 17.4 ft	Z = 24.10

Total Volume	=	135.0 ft^3	85.96 ft^3	70.3 ft^3
Gas Side Volume	=	64.4 ft^3	51.7 ft^3	34.2 ft^3
Air Side Volume	=	64.4 ft^3	31.3 ft^3	34.2 ft^3

Thus if volume were of primary concern to the designer, by using the 17.8 - 3/8 W surface on both sides, he could realize nearly a 50 percent volume reduction from his original design.

To use the Soland method to quantitatively predict the volume savings that would be realized, a new heat exchanger problem was considered. In this problem, one side of the heat exchanger was taken to be identical to the gas side of the original heat-exchanger in the previous problem, the other side of the heat-exchanger was taken to have condensing steam flowing through it. The Plain Plate-Fin 11.1 surface is to be replaced with Wavy-Fin Surface 17.8 - 3/8 W. From Figure 7, V_c/V_o = .46. In other words, a predicted 54% volume saving on the gas side should be realized. Calculated results are as follows:

Original (11.1 Surface)	New (17.8 - 3/8 W Surface)
X = 6.0 ft	(XZ) = 53.44 ft^2
Y = 3.0 ft	Y = .95 ft
Z = 7.5 ft	
Gas Side Volume = 64.04 ft^3	Gas Side Volume = 30.52 ft^3

Actual $\frac{V_a}{V_o}$ = .47 - 53% Actual Volume Savings.

The performance parameters proposed by Soland, $f_n Re_n / D_n^2$ and $j_n Re_n^3 / D_n^4$, allowed us to compare the three surfaces considered and decide which was the "best" without going through a complete set of heat-exchanger design calculations. In the case where literally hundreds of different surfaces are to be considered, the value of a comparison technique such as this can not be understated.

IV. ADDITIONAL SURFACE COMPARISONS

In reference [1], Soland constructed performance parameter plots for most of the surfaces found in reference [2]. He concluded that wavy-fin plate-finned surface 17.8 - 3/8 W was the "best" for the cases considered, as explained in section II.B earlier.

The following surfaces, taken from sources other than reference [2], have been plotted in Figure 8 and a comparison with surface 17.8 - 3/8 W is made.

Figure 8. LEGEND

REFERENCE	SURFACE	CURVE NUMBER
1	17.8 - 3/8 W	1
5	1	5-1
5	2	5-2
10	TPFR 1	10
11	1/8 - 13.95	11-1
11	11.5 - 3/8 W	11-2
11	13.95(P)	11-3

It is noted that the surface 17.8 - 3/8 W is still the "best", but at higher Reynolds numbers its performance is equaled by strip-fin surface 1/8 - 13.95 of reference [11]. Not shown in Figure 8, but if the fluid is changed from air at $90^{\circ}F$ to air at $500^{\circ}F$, strip-fin surface 1/8 - 13.95 becomes superior at the higher Reynolds number.

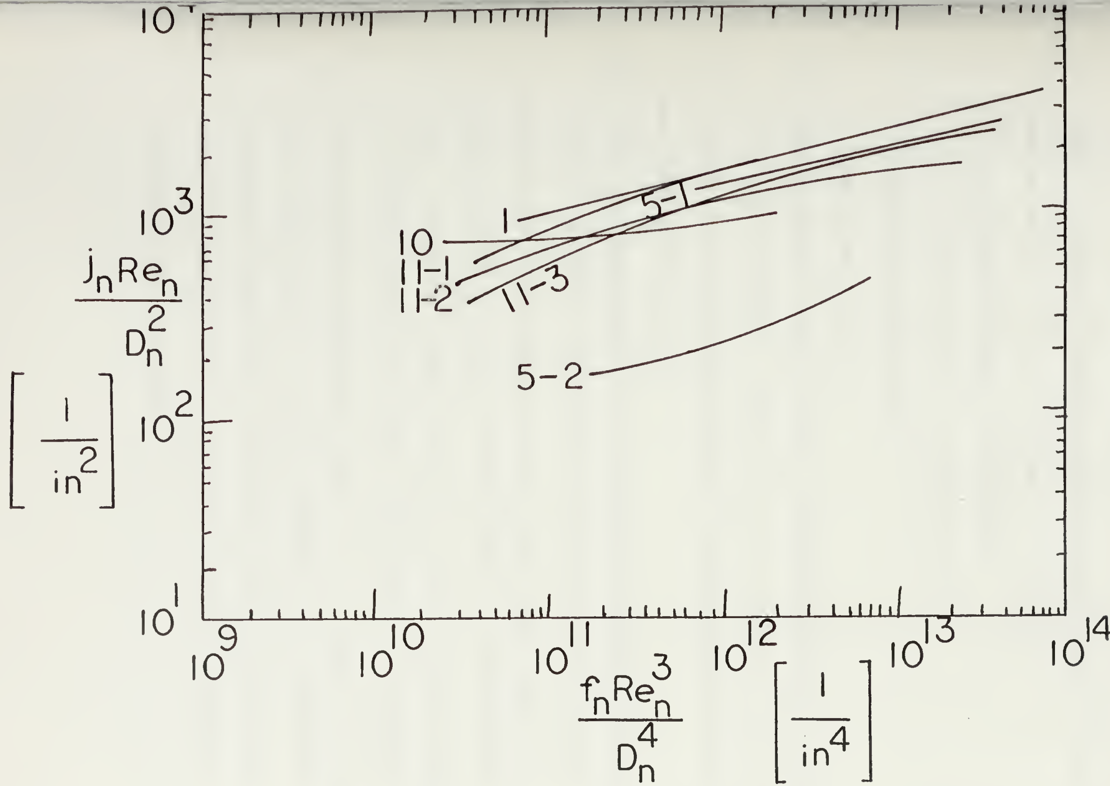


Figure 8. Performance Parameter Curves For Some Additional Surfaces.

V. CONCLUSIONS

1. Soland's comparison technique permits ready comparison of heat transfer surfaces in four different applications:
   a. Same shape and volume heat exchanger
   b. Same exchanger volume and pumping power
   c. Same pumping power and NTU
   d. Same volume and NTU
2. Soland's method should be used to construct performance parameter plots such as Figure 4. The ratio plots of Figure 5 are of little or no value except to demonstrate the possible comparison results available from Figure 4.
3. Unless the constraints of the comparison technique are kept in mind very carefully, the "best" surface may well not be the surface that the designer will end up selecting. The first example problem in section III makes the point that using the "best" surface will reduce volume, but a very long and slender heat exchanger shape will occur. This may not be acceptable.
4. In the numerous applications where the heat transfer resistance on the opposite side of the heat exchanger is not negligible, the predicted improvement will not be fully realized but the comparison is still valid qualitatively and the comparison technique is still a powerful tool.

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APPENDIX I.

SIZING CROSSFLOW PLATE-FINNED HEAT EXCHANGERS FOR A GIVEN JOB

The "job" of the heat exchanger here will be defined as transferring a specified amount of heat between two fluids at given flow rates and with specified amounts of pumping power (i.e., core pressure drop) on each side. This then specifies values for the following quantities:

$$w_1 \; , \; T_{in_1} \; , \; T_{out_1} \; , \; \Delta P_1 \; , \; P_{1_{in}} \qquad \text{(hot)}$$

$$w_2 \; , \; T_{in_2} \; , \; T_{out_2} \; , \; \Delta P_2 \; , \; P_{2_{in}} \qquad \text{(cold)}$$

where subscript 1 refers to the hotter fluid and its associated heat transfer surface and subscript 2 refers to the colder fluid and its heat transfer surface. Core pressure drop will account for by far the greatest portion of total pressure drop because while the addition of fins enhances heat transfer, it also causes greater pressure drops. In the final design, one would have to account for entrance and exit losses as well as core losses.

Figure A-1 shows the heat exchanger arrangement with dimensions X , Y , and Z to be determined.

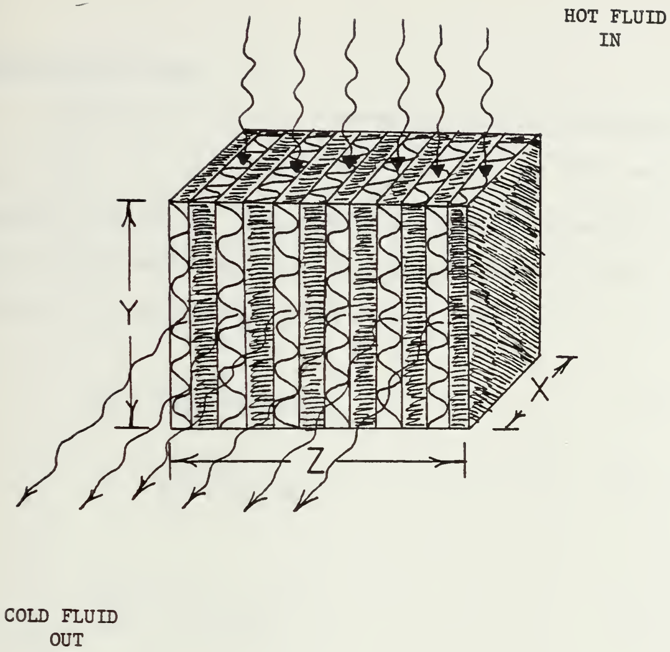

FIGURE A-1.

The Basic Equations

The first portion of the procedure involves determination of A_{c_1} , A_{c_2} , X , Y , and Z based on ΔP_1 and ΔP_2 and an assumed value of G_2. Then, based on heat transfer considerations, it is determined if the proper heat balance exists. If not, another value of G_2 is assumed and the process repeated.

$$\alpha_1 = \frac{b_1 \ \beta_1}{(b_1 + b_2 + 2a)} \tag{1}$$

$$\alpha_2 = \frac{b_2 \ \beta_2}{(b_1 + b_2 + 2a)} \tag{2}$$

$$\frac{A_{c_1}}{A_{f_1}} = \alpha_1 \ r_{h_1} = K_1 \tag{3}$$

$$\frac{A_{c_2}}{A_{f_2}} = \alpha_2 \ r_{h_2} = K_2 \tag{4}$$

$$A_{c_1} = (\alpha_1 \ r_{h_1}) \ (X \cdot Z) = K_1 \ X \ Z \tag{5}$$

$$A_{c_2} = (\alpha_2 \ r_{h_2}) \ (Y \cdot Z) = K_2 \ Y \ Z \tag{6}$$

$$G = \frac{w}{A_c} \tag{7}$$

$$\frac{G_1}{G_2} = \frac{(w_1/A_{c_1})}{(w_2/A_{c_2})} = (\frac{w_1}{w_2}) \ (\frac{A_{c_2}}{A_{c_1}}) \tag{8}$$

From equations (5) and (6)

$$\frac{G_1}{G_2} = \left(\frac{w_1}{w_2}\right) \left(\frac{K_2 \; Y \; Z}{K_1 \; X \; Z}\right) = \left(\frac{w_1}{w_2}\right) \left(\frac{K_2}{K_1}\right) \left(\frac{Y}{X}\right) = K_3 \; \left(\frac{Y}{X}\right) \qquad (9)$$

where $K_3 = \frac{w_1}{w_2} \frac{K_2}{K_1}$

$$\Delta P = \frac{4 \; f \; L \; G^2}{(4 \; r_h) \; (2 \; g_o \; \rho_m)} \qquad (10)$$

$$\frac{G_1^{\;2}}{G_2^{\;2}} = \frac{\dfrac{\Delta P_1 \; r_{h_1} \; 2 \; g_o \; \rho_{m_1}}{f_1 \quad Y}}{\dfrac{\Delta P_2 \; r_{h_2} \; 2 \; g_o \; \rho_{m_2}}{f_2 \quad X}} = \left[\frac{\Delta P_1 \; r_{h_1} \; \rho_{m_1}}{\Delta P_2 \; r_{h_2} \; \rho_{m_2}}\right] \left(\frac{f_2}{f_1}\right) \left(\frac{X}{Y}\right) \qquad (11)$$

Define $K_4 = \left[\frac{\Delta P_1 \quad r_{h_1} \; \rho_{m_1}}{\Delta P_2 \quad r_{h_2} \; \rho_{m_2}}\right]$

From Eq. (9) and (11)

$$\frac{G_1^2}{G_2^2} = K_4 \quad \left(\frac{f_2}{f_1}\right) \left(\frac{X}{Y}\right) = K_3^2 \quad \left(\frac{Y^2}{X^2}\right) \qquad (12)$$

or

$$\frac{K_3^2}{K_4} \quad \left(\frac{Y^3}{X^3}\right) = \frac{f_2}{f_1} = K_5 \quad \left(\frac{Y^3}{X^3}\right) \qquad (13)$$

where $K_5 \equiv \frac{K_3^2}{K_4}$

Eliminate (Y/X) with Eqs. (9) and (13),

$$\frac{f_2}{f_1} = \left(\frac{G_1}{G_2}\right)^3 \frac{K_5}{K_3{}^3} = K_6 \left(\frac{G_1}{G_2}\right)^3 \tag{14}$$

where

$$K_6 = \frac{K_5}{K_3{}^3}$$

Now select a value of Re_2 and calculate G_2 , $(G_2 = Re_2 \mu_2/4\ r_{h_2})$. From the given data read the corresponding value of f_2 and j_2. From equation (14)

$$G_1^3\ f_1 = G_2{}^3\ f_2/K_6$$

or

$$Re_1{}^3\ f_1 = \frac{f_2\ G_2^3\ (4\ r_{h_1})^3}{K_6\ \mu_1{}^3} \tag{15}$$

From the f_1 vs Re_1 data plot for surface 1, determine Re_1 and f_1 to satisfy Eq. (15). If this condition lies outside of the range of the data plot for surface 1 try a new assumption for Re_2 and repeat the calculations. Read also j_1 from the data plot. Calculate G_1 $(G_1 = Re_1\ \mu_1/4\ r_{h_1})$.

From equation (10) calculate X and Y:

$$X = \frac{(\Delta P_2)\ (r_{h_2})\ (2\ g_o\ \rho_{m_2})}{f_2\ \ G_2{}^2} \tag{16a}$$

$$Y = \frac{(\Delta P_1)\ (r_{h_1})\ (2\ g_o\ \rho_{m_1})}{f_1\ \ G_1^{\ 2}} \tag{16 b}$$

and from equations (5) and (7) calculate Z:

$$Z = \frac{A_{c_1}}{K_1\ X} = \frac{(w_1/G_1)}{K_1\ \ X} \tag{17}$$

$$h_1 = j_1\ C_{pm_1}\ G_1\ \ Pr_1^{\ -2/3} \tag{18a}$$

$$h_2 = j_2\ C_{pm_2}\ G_2\ Pr_2^{\ -2/3} \tag{18b}$$

$$m_1 = \frac{2h_1}{\delta_1\ k_1} \tag{19a}$$

$$m_2 = \frac{2h_2}{\delta_2\ k_2} \tag{19b}$$

$$\eta_{f_1} = \frac{\tanh\ (m_1 \ell_1)}{m_1 \ell_1} \tag{20a}$$

$$\eta_{f_2} = \frac{\tanh\ (m_2 \ell_2\ )}{m_2 \ell_2} \tag{20b}$$

$$\eta_{o_1} = 1 - \left(\frac{A_{fin}}{A_T}\right)_1 \ \ (1 - \eta_{f_1}) \tag{21a}$$

$$\eta_{o_2} = 1 - \left(\frac{A_{fin}}{A_T}\right)_2 (1 - \eta_{f_2}) \tag{21b}$$

$$\frac{1}{A\,U} = \frac{1}{A_{T_1}\,U_1} = \frac{1}{A_{T_2}\,U_2} = \frac{1}{A_{T_1}\,\eta_{o_1}\,h_1} + \frac{1}{A_{T_2}\,\eta_{o_2}\,h_2} \tag{22}$$

where $A_{T_1} = \alpha_1\,XYZ$

$A_{T_2} = \alpha_2\,XYZ$

If $C_1 < C_2$, $C_{min} = C_1$, $C_{min}/C_{max} = C_1/C_2$

$$\varepsilon = \frac{T_{in_1} - T_{out_1}}{T_{in_1} - T_{in_2}} \tag{23a}$$

If $C_2 < C_1$, $C_{min} = C_2$, $C_{min}/C_{max} = C_2/C_1$

$$\varepsilon = \frac{T_{out_2} - T_{in_2}}{T_{in_1} - T_{in_2}} \tag{23b}$$

Also

$$NTU = \frac{AU}{C_{min}} \tag{24}$$

From Figure A-2 with C_{min}/C_{max} and the NTU magnitude calculated from Equation (24) read the magnitude of ε which would result for the assumed Re_2. If this ε is not the desired

magnitude assume a different magnitude of Re_2 and repeat the calculation.

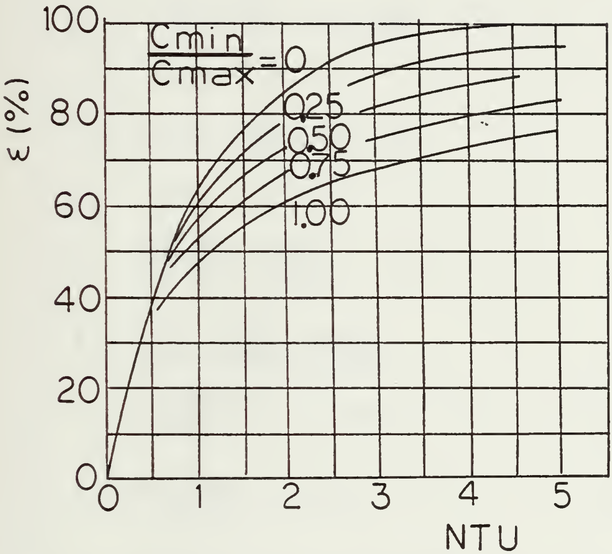

Figure A-2.

APPENDIX II.

SIZING CROSSFLOW PLATE-FINNED HEAT EXCHANGER FOR A GIVEN JOB

CALCULATIONS

$$\alpha_1 = \frac{(.25)\ (367)}{(.25 + .25 + (2)\ (.012))} = 175.1 \qquad (1)$$

$$\alpha_2 = 122.1 \qquad (2)$$

$$K_1 = (175.1)\ (.00253) = .4430 \qquad (3)$$

$$K_2 = .4458 \qquad (4)$$

$$K_3 = \frac{(195{,}895)\ (.4458)}{(193{,}000)\ (.4430)} = 1.0214 \qquad (9)$$

$$K_4 = \frac{(.42)\ (.00253)\ (.0362)}{(.54)\ (.00365)\ (.3565)} \qquad (11)$$

$$K_5 = \frac{(1.0214)^2}{(.0547)} = 19.057 \qquad (13)$$

$$K_6 = \frac{(19.057)}{(1.0214)^3} = 17.8843 \qquad (14)$$

The calculations for the first selected Re_2 will be shown. Succeeding iteration results are tabulated at the end. Select

$Re_2 = 2000$

$$G_2 = \frac{(2000)\ (.069)}{(4)\ (.00365)} = 9452\ \text{lb/hr-ft}^2$$

from plotted data for surface 2

$f_2 = .0426$ $\qquad$ $j_2 = .0090$

$$Re_1^3 \; f_1 = \frac{(.0426)\;(9452)^3\;(4 \cdot .00253)^3}{(17.8843)\;(.073)^3} = 5.36 \times 10^6 \qquad (15)$$

from plotted data for surface 1 and equation (15)

$Re_1 = 550$ $\qquad$ $f_1 = .033$ $\qquad$ $j_1 = .0078$

$$G_1 = \frac{(550)\;(.073)}{(4)\;(.00253)} = 3967 \text{ lb/hr-ft}^2$$

$$X = \frac{(.54)\;(.00365)\;(2)\;(32.2)\;(.3565)\;(3600)^2\;(144)}{(.0426)\;(9452)^2} = 22.2 \text{ ft} \qquad (15a)$$

$$Y = 8.9 \text{ ft} \qquad (15b)$$

$$Z = \frac{(195,895)}{(.4430\;(3967)\;(22.2)} = 5.0 \text{ ft} \qquad (17)$$

$$h_1 = (.0078)\;(.259)\;(3967)\;(.66)^{-2/3} = 10.57 \text{ BTU/(hr-ft}^2\text{-}^{o}\text{F-ft)} \qquad (18a)$$

$$h_2 = 28.17 \text{ BTU/(hr-ft}^2\text{-}^{o}\text{F/ft)} \qquad (18b)$$

$$m_1 = \sqrt{\frac{(2)\;(10.57)}{(\frac{.006}{12})\;(12)}} = 59.4/\text{ft} \qquad (19a)$$

$$m_2 = 96.9/\text{ft} \qquad (19b)$$

$$\eta_{f_1} = \frac{\tanh\;(59.4)\;(\frac{.25}{24})}{(59.4)\;(\frac{.25}{24})} = .8894 \qquad (20a)$$

$$\eta_{f_2} = .7584 \qquad (20b)$$

$$\eta_{o_1} = 1 - (.756)\ (1 - .8894) = .9164 \qquad (21a)$$

$$\eta_{o_2} = .8454 \qquad (21b)$$

$$\frac{1}{AU} = \frac{1}{(175.1)\ (22.2)\ (8.9)\ (5.0)\ (.9164)}$$

$$+ \frac{1}{(122.1)\ (22.2)\ (8.9)\ (5.0)\ (28.17)\ (.8454)}$$

$$AU = 1,062,032$$

$$C_1 = (195,895)\ \ (.259) = 50.737\ \text{BTU/hr-}^{o}\text{F}$$

$$C_2 = (193,000)\ (.251) = 48,443\ \text{BTU/hr-}^{o}\text{F}$$

$$\frac{C_{min}}{C_{max}} = \frac{48,443}{50,737} = .955$$

$$\varepsilon = \frac{691 - 347}{805 - 347} = .751 \qquad (23b)$$

$$NTU = \frac{1,062,032}{48,443} = 21.9$$

From Figure 2A with $C_{min}/C_{max} = .955$ and $NTU = 21.9$, $\varepsilon = .78$ which is larger than the required .751 from equation (23). The resulting calculations from succeeding iterations are tabulated below.

CALCULATION RESULTS FOR VARIOUS Re_2

(Required ε = .751)

Re_2	2000	5000	3500	4090
G_2	9452	23,630	16,541	19,329
Re_1	550	1700	1200	1370
G_1	3967	12,263	8656	9882
X	22.2	4.2	8.1	6.0
Y	8.9	2.2	3.5	3.0
Z	5.0	8.7	6.3	7.5
NTU	21.9	2.97	5.33	4.26
ε	.78	.70	.76	.75

The Re_2 required to satisfy the design specifications of heat transfer and pressure drop in 4090.

It appears that in the trial and error process of solution if after assuming a magnitude for Re_2 , the resulting ε is higher than the required magnitude, the assumed Re_2 should be increased.

APPENDIX III.

SURFACE DATA FOR FIGURE 8.

Curve 1

Type	Wavy Fin
Fin Pitch	17.8 per inch
b	.413 in.
σ	.006 in.
A_{fin}/A_T	.892
β	514 ft^2/ft^3
$4\ r_h$	.00696 ft

Re	j	f
5000	.00675	.0293
4000	.00740	.0320
3000	.00835	.0358
2000	.00982	.0421
1000	.0129	.0579
600	.0158	.0738

Curve 5-1

Type	Plain Fin
Fin Pitch	10 per inch
b	1.204 in.
σ	.012 in.
A_{fin}/A_T	.937
β	272.7 ft^2/ft^3
$4\ r_h$	.01184 ft

Re	j	f
4410	.00549	.0136
2880	.00647	.0153
2200	.00743	.0169
1224	.0101	.0238
887	.0124	.0284

Curve 5-2

Type	Plain Fin
Fin Pitch	9 per inch
b	.782 in.
σ	.008 in.
A_{fin}/A_T	.890
β	259.7 ft^2-ft^3
$4\ r_h$	.01342

Re	j	f
6220	.00380	.00814
5390	.00388	.00846
4690	.00391	.00861
3750	.00404	.00926
2530	.00409	.0106

Curve 10

Type	Perforated (Circular)
Fin Pitch	7.25 per inch
b	.0878 in.
σ	.0028 in.
A_{fin}/A_T	.917
β	667 ft^2/ft^3
$4\ r_h$	.0055 ft.

Re	j	f
1000	.004	.0149
600	.006	.0220
400	.009	.0360
200	.017	.0699

Curve 11-1

Type	Strip-Fin
Fin Pitch	13.95 per inch
b	.375 in.
σ	.01 in.
A_{fin}/A_T	.84
β	381 ft^2/ft^3
$4\ r_h$	.00879 ft

Re	j	f
6000	.01110	.0650
4000	.01250	.0684
2000	.0155	.0765
1000	.0192	.0927
500	.0233	.131

Curve 11-2

Type	Wavy-Fin
Fin Pitch	11.5 per inch
b	.375 in.
σ	.010 in.
A_{fin}/A_T	.822
β	347 ft^2/ft^3
$4\ r_h$	.00993

Re	j	f
8000	.00746	.0357
5000	.00890	.0427
2000	.0126	.0625
1000	.0158	.0845
500	.0185	.1111

Curve 11-3

Type	Perforated (circular)
Fin Pitch	13.95 per inch
b	.2 in.
σ	.012 in.
A_{fin}/A_T	.705
β	381 ft^2/ft^3
$4\ r_h$	.00822 ft

Re	j	f
8000	.00547	.0129
5000	.00631	.0146
2000	.00700	.0187
1000	.00893	.0232
500	.0132	.0407